WINTER– 19 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code: 22224**

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.		**Solve any FIVE of the following:**	**10**
	a)	State whether the function is odd or even, $f(x)=\frac{e^x+e^{-x}}{2}$	**02**
	Ans	$f(x)=\frac{e^x+e^{-x}}{2}$	
		$\therefore f(-x)=\frac{e^{-x}+e^{-(-x)}}{2}$	½
		$\therefore f(-x)=\frac{e^{-x}+e^{x}}{2}$	½
		$\therefore f(-x)=f(x)$	½
		$\therefore$ function is even.	½
		--	
	b)	If $f(x)=\log_4 x+3$, find $f\left(\frac{1}{4}\right)$	**02**
	Ans	$f(x)=\log_4 x+3$	
		$f\left(\frac{1}{4}\right)=\log_4\left(\frac{1}{4}\right)+3$	1
		$=-\log_4 4+3$	
		$=-1+3=2$	1

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Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	c)	Find $\frac{dy}{dx}$ if $y = x^2.e^x$	**02**
	Ans	$y = x^2.e^x$	
		$\frac{dy}{dx} = x^2.e^x + e^x.2x$	1
		$\frac{dy}{dx} = xe^x\left(x+2\right)$	1
		--	
	d)	Evaluate $\int\left[e^x + a^x + x^a + a^a\right]\,dx$	**02**
	Ans	$\int\left[e^x + a^x + x^a + a^a\right]\,dx$	
		$= e^x + \frac{a^x}{\log a} + \frac{x^{a+1}}{a+1} + a^a x + c$	2
		--	
	e)	Evaluate: $\int\left[\frac{1}{1+\cos 2x}\right]dx$	**02**
	Ans	$\int\left[\frac{1}{1+\cos 2x}\right]dx$	
		$= \int\left[\frac{1}{2\cos^2 x}\right]dx$	1
		$= \frac{1}{2}\int \sec^2 x dx$	
		$= \frac{1}{2}\tan x + c$	1
		--	
	f)	Find the area bounded by $y = x$, X – axis and $x = 0$ to $x = 4$.	**02**
	Ans	Area $A = \int_a^b y\,dx$	
		$= \int_0^4 x dx$	½
		$= \left[\frac{x^2}{2}\right]_0^4$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.		$=\left(\frac{4^2}{2}-0\right)$	½
		$=8$	½
		--	
	g)	Find a real root of the equation $x^3+4x-9=0$ in the interval $(1,2)$ by using Bisection method. (only one iteration)	**02**
	Ans	Let $f(x)=x^3+4x-9$	
		$f(1)=-4$	1
		$f(2)=7$	
		$\therefore$ the root is in $(1,2)$	
		$x_1=\frac{a+b}{2}=\frac{1+2}{2}=1.5$	1
		--	
2		**Solve any <u>THREE</u> of the following:**	**12**
	a)	Find $\frac{dy}{dx}$, if $y=\frac{5e^x}{3e^x+1}$ at $x=0$	**04**
	Ans	$y=\frac{5e^x}{3e^x+1}$	
		$\frac{dy}{dx}=\frac{(3e^x+1)5e^x-5e^x(3e^x)}{(3e^x+1)^2}$	2
		$\frac{dy}{dx}=\frac{15e^{2x}+5e^x-15e^{2x}}{(3e^x+1)^2}$	
		$\frac{dy}{dx}=\frac{5e^x}{(3e^x+1)^2}$	1
		$at\ x=0$	
		$\frac{dy}{dx}=\frac{5e^0}{(3e^0+1)^2}$	
		$=\frac{5}{16}$ or 0.3125	1
		--	

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Q. No.	Sub Q. N.	Answers	Marking Scheme
2.	b)	If $x=a\left(1+\cos\theta\right)$, $y=a\left(1-\cos\theta\right)$, find $\frac{dy}{dx}$	**04**
	Ans	$x=a\left(1+\cos\theta\right)$, $y=a\left(1-\cos\theta\right)$	
		$\frac{dx}{d\theta}=a\left(-\sin\theta\right)=-a\sin\theta$, $\frac{dy}{d\theta}=a\left(0+\sin\theta\right)=a\sin\theta$	1+1
		$\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{a\sin\theta}{-a\sin\theta}$	1
		$\frac{dy}{dx}=-1$	1
	c)	A metal wire 36 cm long is bent to form a rectangle. Find its dimensions when its area is maximum.	**04**
	Ans	Let length of rectangle $=x$, breadth $=y$	
		$\therefore 2x+2y=36$	
		$\therefore y=18-x$	1
		Area $A=x\times y$	
		$A=x\left(18-x\right)$	1
		$\therefore A=18x-x^2$	
		$\therefore \frac{dA}{dx}=18-2x$	½
		$\therefore \frac{d^2A}{dx^2}=-2$	½
		Let $\frac{dA}{dx}=0$	
		$\therefore 18-2x=0$	
		$\therefore x=9$	½
		at $x=9$	
		$\frac{d^2A}{dx^2}=-2<0$	
		Area is maximum at $x=9$	
		Length $=9$; breadth $=9$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	d)	Find radius of curvature of a curve $y=\log(\sin x)$ at $x=\pi/2$	**04**
	Ans	$y=\log(\sin x)$	
		$\therefore \frac{dy}{dx}=\frac{1}{\sin x}.\cos x=\cot x$	½
		$\therefore \frac{d^2y}{dx^2}=-\cos ec^2 x$	½
		at $x=\pi/2$	
		$\frac{dy}{dx}=\cot\frac{\pi}{2}=0$	½
		$\frac{d^2y}{dx^2}=-\cos ec^2\frac{\pi}{2}=-1$	½
		$\therefore$ Radius of curvature is, $\rho=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$	
		$\therefore \rho=\frac{\left[1+(0)^2\right]^{\frac{3}{2}}}{-1}$	1
		$\therefore \rho=-1$ i.e. 1	1
		--	
3.		**Solve any THREE of the following:**	**12**
	a)	Find equation of the tangent and normal to the curve $4x^2+9y^2=40$ at point $(1,2)$	**04**
	Ans	$4x^2+9y^2=40$	
		$\therefore 8x+18y\frac{dy}{dx}=0$	½
		$\therefore \frac{dy}{dx}=\frac{-8x}{18y}$	
		$\therefore \frac{dy}{dx}=\frac{-4x}{9y}$	½
		at $(1,2)$	
		$\therefore \frac{dy}{dx}=\frac{-4(1)}{9(2)}$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	a)	$\therefore \frac{dy}{dx}=\frac{-2}{9}$	
		$\therefore$ slope of tangent , $m=\frac{-2}{9}$	½
		Equation of tangent at $(1,2)$ is	
		$y-2=\frac{-2}{9}(x-1)$	½
		$\therefore 9y-18=-2x+2$	
		$\therefore 2x+9y-20=0$	½
		$\therefore$ slope of normal , $m'=\frac{-1}{m}=\frac{9}{2}$	½
		Equation of normal at $(1,2)$ is	
		$y-2=\frac{9}{2}(x-1)$	½
		$\therefore 2y-4=9x-9$	
		$\therefore 9x-2y-5=0$	½
		--	
	b)	Find $\frac{dy}{dx}$ if $y=\tan^{-1}\left[\frac{2x}{1+35x^2}\right]$	**04**
	Ans	$y=\tan^{-1}\left[\frac{7x-5x}{1+7x.5x}\right]$	1
		$y=\tan^{-1}7x-\tan^{-1}5x$	1
		$\frac{dy}{dx}=\frac{7}{1+49x^2}-\frac{5}{1+25x^2}$	2
		--	
	c)	If $x^y=e^{x-y}$ Show that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$	**04**
	Ans	$x^y=e^{x-y}$	
		$\log x^y=\log e^{x-y}$	½
		$y\log x=x-y\log e$	½
		$y\log x=x-y$	
		$y\log x+y=x$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	c)	$y(\log x+1)=x$	
		$y=\dfrac{x}{\log x+1}$	1
		$\dfrac{dy}{dx}=\dfrac{(\log x+1).1-x.\frac{1}{x}}{(\log x+1)^2}$	1
		$=\dfrac{\log x}{(\log x+1)^2}$	½
		--	
	d)	Evaluate $\int\dfrac{dx}{5+3\cos 2x}$	**04**
	Ans	$\int\dfrac{dx}{5+3\cos 2x}$	
		Put $\tan x=t$, $dx=\dfrac{dt}{1+t^2}$	
		$\cos 2x=\dfrac{1-t^2}{1+t^2}$	
		$\int\dfrac{\frac{dt}{1+t^2}}{5+3\left(\frac{1-t^2}{1+t^2}\right)}$	1
		$=\int\dfrac{dt}{5(1+t^2)+3(1-t^2)}$	
		$=\int\dfrac{dt}{5+5t^2+3-3t^2}$	
		$=\int\dfrac{dt}{2t^2+8}$	1
		$=\int\dfrac{dt}{(\sqrt{2}t)^2+(\sqrt{8})^2}$ OR $=\dfrac{1}{2}\int\dfrac{dt}{t^2+4}$	
		$=\dfrac{1}{\sqrt{8}}\tan^{-1}\left(\dfrac{\sqrt{2}t}{\sqrt{8}}\right).\dfrac{1}{\sqrt{2}}+c$ $=\dfrac{1}{2}.\dfrac{1}{2}\tan^{-1}\left(\dfrac{t}{2}\right)+c$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	d)	$=\frac{1}{\sqrt{16}}\tan^{-1}\left(\frac{\sqrt{2}\tan x}{\sqrt{8}}\right)+c$ OR $=\frac{1}{4}\tan^{-1}\left(\frac{t}{2}\right)+c$	½
		$=\frac{1}{4}\tan^{-1}\left(\frac{\tan x}{2}\right)+c$ OR $=\frac{1}{4}\tan^{-1}\left(\frac{\tan x}{2}\right)+c$	½
		--	
4.		**Solve any <u>THREE</u> of the following:**	**12**
	a)	Evaluate $\int\frac{\left[e^{x}(x+1)\right]}{\cos^{2}(x.e^{x})}dx$	**04**
	Ans	$\int\frac{\left[e^{x}(x+1)\right]}{\cos^{2}(x.e^{x})}dx$	
		Put $x.e^{x}=t$	
		$\therefore(x.e^{x}+e^{x}.1)dx=dt$	
		$\left[e^{x}(x+1)\right]dx=dt$	1
		$\therefore\int\frac{1}{\cos^{2}t}dt$	1
		$=\int\sec^{2}t\ dt$	1
		$=\tan t+c$	½
		$=\tan(x.e^{x})+c$	½
		--	
	b)	Evaluate : $\int\frac{dx}{2x^{2}+3x+2}$	**04**
	Ans	$\int\frac{dx}{2x^{2}+3x+2}$	
		$=\frac{1}{2}\int\frac{dx}{x^{2}+\frac{3}{2}x+1}$	½
		$==\frac{1}{2}\int\frac{dx}{x^{2}+\frac{3}{2}x+\frac{9}{16}+1-\frac{9}{16}}$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	b)	$=\frac{1}{2}\int\frac{dx}{\left(x+\frac{3}{4}\right)^2+\frac{7}{16}}$	½
		$=\frac{1}{2}\int\frac{dx}{\left(x+\frac{3}{4}\right)^2+\left(\frac{\sqrt{7}}{4}\right)^2}$	1
		$=\frac{1}{2}\frac{1}{\frac{\sqrt{7}}{4}}\tan^{-1}\left(\frac{x+\frac{3}{4}}{\frac{\sqrt{7}}{4}}\right)+c$	1
		$=\frac{2}{\sqrt{7}}\tan^{-1}\left(\frac{4x+3}{\sqrt{7}}\right)+c$	
	c)	Evaluate $\int x^2.\tan x\ dx$	**04**
	Ans	$\int x^2.\tan x\ dx$	
		$=x^2\left(\int\tan x dx\right)-\int\left(\int\tan x dx.\frac{d}{dx}\left(x^2\right)\right)dx$	½
		$=x^2\log(\sec x)-\int\log(\sec x).2x\ dx$	1
		$=x^2\log(\sec x)-2\left[\log(\sec x)\frac{x^2}{2}-\int\frac{1}{\sec x}.\sec x\tan x.\frac{x^2}{2}dx\right]$	½
		$=x^2\log(\sec x)-2\left[\log(\sec x)\frac{x^2}{2}-\frac{1}{2}\int x^2.\tan x.dx\right]$	
		$=x^2\log(\sec x)-2\left[\log(\sec x)\frac{x^2}{2}-\frac{1}{2}I\right]$	½
		$I=x^2\log(\sec x)-\log(\sec x)x^2+I$	½
		Note: If students attempted to solve the question give appropriate marks.	
	d)	Evaluate $\int\frac{\sec^2 x}{(\tan x)(\tan x+1)}dx$	**04**

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4.	Ans	$\int \frac{\sec^2 x}{(\tan x)(\tan x+1)} dx$	
		Put $\tan x = t$	1
		$\therefore \sec^2 x\, dx = dt$	
		$\therefore \int \frac{1}{(t)(t+1)} dt$	
		$\frac{1}{(t)(t+1)} = \frac{A}{t} + \frac{B}{t+1}$	
		$\therefore 1 = A(t+1) + B(t)$	
		$\therefore$ Put $t = 0$, $A = 1$	½
		Put $t = -1$, $B = -1$	½
		$\therefore \frac{1}{(t)(t+1)} = \frac{1}{t} - \frac{1}{t+1}$	
		$\therefore \int \frac{1}{(t)(t+1)} dt = \int \left(\frac{1}{t} - \frac{1}{t+1}\right) dt$	½
		$= \log(t) - \log(t+1) + c$	1
		$= \log(\tan x) - \log(\tan x + 1) + c$	½
		--	
	e)	Evaluate: $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\tan x}} dx$	**04**
	Ans	$\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\tan x}} dx$	
		$= \int_0^{\frac{\pi}{2}} \frac{1}{1+\frac{\sqrt{\sin x}}{\sqrt{\cos x}}} dx$	
		Let I= $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx$ --------------------(1)	

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	e)	$I=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}dx$	1
		$I=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ ----------------------------(2)	1
		Add (1) and (2)	
		$I+I=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx+\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$	1
		$2I=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$	
		$2I=\int_0^{\frac{\pi}{2}} dx$	
		$2I=\left[x\right]_0^{\frac{\pi}{2}}$	½
		$2I=\frac{\pi}{2}-0$	
		$\therefore I=\frac{\pi}{4}$	½
		--	
5		**Solve any <u>TWO</u> of the following:**	**12**
	a)	Find area bounded by the curve $y=x^2$ and the line $y=x$	**06**
	Ans	We have $y=x^2$ and $y=x$	
		$\therefore x^2-x=0$	
		$\therefore x(x-1)=0$	
		$\therefore x=0$ or $x=1$	1
		Area $=\int_a^b (y_1-y_2)\,dx$	
		$=\int_0^1 (x^2-x)\,dx$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	a)	$=\left[\frac{x^3}{3}-\frac{x^2}{2}\right]_0^1$	1
		$=\left[\frac{1^3}{3}-\frac{1^2}{2}-0\right]$	1
		$=-\frac{1}{6}$	1
		$\therefore A=\frac{1}{6}$ or 0.167 $(\because$ Area is always $+ve)$	1
		--	
	b)	Attempt the following:	**06**
	i)	From the differential equation by eliminating the arbitray constant if $y=A\cos x+B\sin x.$	**03**
	Ans	$y=A\cos x+B\sin x.$	
		$\frac{dy}{dx}=-A\sin x+B\cos x$	1
		$\frac{d^2y}{dx^2}=-A\cos x-B\sin x$	
		$=-\left(A\cos x+B\sin x\right)$	1
		$=-y$	
		$\frac{d^2y}{dx^2}+y=0$	1
		--	
	ii)	Solve $\left(1+x^2\right)dy-x^2.ydx=0$	**03**
	Ans	$\left(1+x^2\right)dy-x^2.ydx=0$	
		$\left(1+x^2\right)dy=x^2.ydx$	
		$\frac{dy}{y}=\frac{x^2dx}{1+x^2}$	
		$\int\frac{dy}{y}=\int\frac{x^2dx}{1+x^2}$	1
		$\int\frac{dy}{y}=\int\frac{1+x^2-1dx}{1+x^2}$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.		$\int \frac{dy}{y} = \int \left[1-\frac{1}{1+x^2}\right]dx$	1
		$\log y = x - \tan^{-1} x + c$	1
		--	
	c)	Solve the D.E $\frac{dq}{dt}+\frac{1}{RC}q=\frac{E}{R}$ given that $q=0$ when $t=0$ and E,R,C are constant	**06**
	Ans	$\frac{dq}{dt}+\frac{1}{RC}q=\frac{E}{R}$	
		$I.F = e^{\int \frac{1}{RC}dt}$	
		$= e^{\frac{t}{RC}}$	1
		$\therefore q.e^{\frac{t}{RC}} = \int \frac{E}{R}.e^{\frac{t}{RC}}dt$	1
		$= \frac{E}{R}e^{\frac{t}{RC}}.\frac{1}{\frac{1}{RC}}+c_1$	1
		$q.e^{\frac{t}{RC}} = e^{\frac{t}{RC}}EC + c_1$	
		given that $q=0$ when $t=0$	
		$0 = e^0 EC + c_1$	1
		$c_1 = -EC$	1
		$q.e^{\frac{t}{RC}} = e^{\frac{t}{RC}}EC - EC$	
		$q = EC\left(1-e^{-\frac{t}{RC}}\right)$	1
		--	
6.		**Solve any <u>TWO</u> of the following:**	**12**
	a)	Attempt the following:	**06**
	i)	Solve the equations by Gauss-Seidal method. (two iterations only)	
		$10x+y+2z=13$, $3x+10y+z=14$, $2x+3y+10z=15$	**03**

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	a)(i)	$10x+y+2z=13$, $3x+10y+z=14$, $2x+3y+10z=15$	
		$x=\frac{1}{10}(13-y-2z)$ $y=\frac{1}{10}(14-3x-z)$ $z=\frac{1}{10}(15-2x-3y)$	1
		Starting with $x_0=y_0=z_0=0$	
		$x_1=1.3$ $y_1=1.01$ $z_1=0.937$	1
		$x_2=1.012$ $y_2=1.003$ $z_2=0.997$	1
	(ii)	Solve the following system of equation by using Jacobi-Iteration method. (two iterations) $5x+2y+z=12$, $x+4y+2z=15$, $x+2y+5z=20$	**03**
	Ans	$5x+2y+z=12$ $x+4y+2z=15$ $x+2y+5z=20$	
		$x=\frac{1}{5}(12-2y-z)$ $y=\frac{1}{4}(15-x-2z)$ $z=\frac{1}{5}(20-x-2y)$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	a)(ii)	Starting with $x_0 = y_0 = z_0 = 0$	
		$x_1 = 2.4$ $y_1 = 3.75$ $z_1 = 4$	1
		$x_2 = 0.1$ $y_2 = 1.15$ $z_2 = 2.02$	1
	b)	Solve the following system of equations by using Gauss elimination method. $x+2y+3z=14, 3x+y+2z=11, 2x+3y+z=11$	**06**
	Ans	$3x+6y+9z=42$ $3x+y+2z=11$ $-$ $5y+7z=31$ and $2x+4y+6z=28$ $2x+3y+z=11$ $-$ $y+5z=17$	1+1
		$5y+25z=85$ $5y+7z=31$ $-$ $18z=54$	1
		$\therefore z=3$	1
		$y=2$	1
		$x=1$	1
	c)	Using Newton-Raphson method find the approximate root of the equation $x^2+x-5=0$ (use four iterations)	**06**
	Ans	$f(x)=x^2+x-5$ $f(1)=-3<0$ $f(2)=1>0$	1
		$f'(x)=2x+1$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	c)	Initial root $x_0=2$	
		$\therefore f'(2)=5$	
		$x_1=x_0-\frac{f(x_0)}{f'(x_0)}=2-\frac{f(2)}{f'(2)}=1.8$	1
			1
		$x_2=1.8-\frac{f(1.8)}{f'(1.8)}=1.7913$	
		$x_3=1.7913-\frac{f(1.7913)}{f'(1.7913)}=1.7912$	1
		$x_4=1.7912-\frac{f(1.7912)}{f'(1.7912)}=1.7912$	1
		OR	
		$Let\ f(x)=x^2+x-5$	
		$f(1)=-3<0 \quad f(2)=1>0$	1
		$f'(x)=2x+1$	1
		Initial root $x_0=2$	
		$x_i=x-\frac{f(x)}{f'(x)}=x-\frac{x^2+x-5}{2x+1}$	
		$=\frac{2x^2+x-x^2-x+5}{2x+1}$	
		$=\frac{x^2+5}{2x+1}$	2
		$x_1=1.8$	½
		$x_2=1.7913$	½
		$x_3=1.7912$	½
		$x_4=1.7912$	½
		--	
		Important Note	
		In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.	